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The corollary is if also f is conformal transformation of D with conformal inverse f^{-1}, then $f(z) = az$ and $|a| = 1$. Schwarz Lemma is that $|f(z)| \leq 1$ and $f(0) = 0$ then $|f(z)| \leq |z|$ and $|f'(0)| \leq 1$.

Apply Schwarz lemma to f and to f^{-1}.

$$|f(z)| = |z|$$

$$|(f^{-1})'(0)| \leq 1$$

$$(ff')'(0) = 1$$

$$\therefore |f'(0)| = 1$$

$$\therefore f(z) = az$$

Theorem we tried to see in Friday, a conformal transformation of D onto D have the form,

$$f(z) = \omega\phi_a(z)$$

$$\phi_a(z) = \frac{z-a}{1-\overline{a}z}$$

$$|w| = 1 \quad |a| < 1$$

Proof is that suppose f is a conformal transformation of D then $f(0) = a$.

Definition 1. A C^2 function on $\mathbb{C}$ is said harmonic if it satisfies the Laplace's Equation.

$$u_{xx} + u_{yy} = 0$$

For us, the prime examples are real parts and imaginary parts of Holomorphic functions.

$$u_x = v_y$$

$$u_y = -v_x$$