Computational Complex Analysis : : Class 25

March 6, 2024

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For the case of $N = 1$

$$\int_{C_{z_0}} \frac{1}{2\pi i} \frac{f'(s)}{f(s) - w} \mathrm{d}s = 1$$

For w near w_0. Now think about another holomorphic function $g(z)$ defined near z_0. Then let's calculate the integral $\frac{1}{2\pi i} \int \frac{f'(s)g(s)}{f(s)-w} \mathrm{d}s$ using the residue theorem. Hence it' is the sum of residues of $\frac{f'(z)g(z)}{f(z)-w}$ inside C_0. This single residue occurs when $f(z) = w$.

$$= \frac{f'(z)g(z)}{f'(z)} = g(z)$$

Here $w = f(z)$. In other words z is what you get by taking w and finding out $f'(w)$.

Now apply the formula with $g(z) = z$.

$$z = \frac{1}{2\pi i} \int_{C_0} \frac{f'(s)s}{f(s) - w} \mathrm{d}s$$

$$f^{-1}(w) = \frac{1}{2\pi i} \int_{C_0} \frac{f'(s)s}{f(s) - w} \mathrm{d}s$$

— Same picture as Monday, let a holomorphic function be $f(z)$ that tends to 0 quadratically as $|z| \to \infty$.

$$\left| f(z) \cdot z^2 \right| \leq C \text{ for large } z$$

We want to apply residue theorem to

$$\pi \cot \pi z f(z)$$

We choose this to be the function because it has poles to be the integers (i am writing what?)

Prove that $\pi \cot \pi z$ is bounded on this path independent of N. For the vertical lines

$$|\cot \pi(N + \frac{1}{2} + iy)| = |\cot(\pi N + \frac{\pi}{2} + iy)| = |\cot(\pi/2 + iy)| = |\tan iy|$$

$$= \frac{|\sinh y|}{\cosh y} < 1$$

(discontinued a portion because I understand nothing what he did) Result: Apply residue theorem then let $N \to \infty$. The integral of this huge path is

$$0 = \sum \operatorname{Res} \left(\pi \cos \pi z f(z) \right)$$

Application of this very same thing.

$$f(z) = \frac{1}{z^{2k}}$$

We will calculate

$$\sum_{-\infty}^{\infty} \frac{1}{n^{2k}}$$

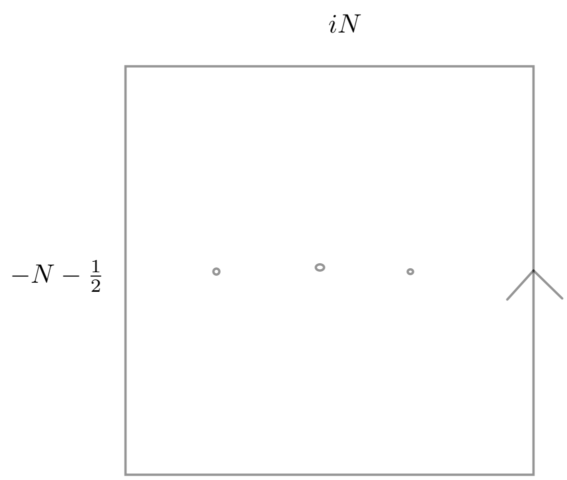

Figure 1: class25-figure-box