Computational Complex Analysis : : Class 16

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Ahmed Saad Sabit, Rice University

$$f(z) = \ldots + \frac{c_{-2}}{(z - z_0)^2} + \frac{c_{-1}}{z - z_0} + c_0 + c_1(z - z_0)$$

$$\text{Res}(f, z_0)$$

also it equals $\frac{1}{2\pi i}\int_C f(z)\mathrm{d}z$. Also it equals the unique $a \in \mathbb{C}$ such that $f(z) - \frac{a}{z - z_0}$.

$$\text{Res}\left((z - z_0)^n, z_0\right) = 1$$

If $n = -1$ otherwise 0.

$$C_1 \text{Res}\left(e^{\frac{1}{z}}, 0\right) = 1$$

Simple pole at z_0

$$f(z) = \frac{c_a}{z - z_0} + c_0 + c_1(z - z_0) + \cdots$$

Suppose our function can be written as a quotient:

$$f(z) = \frac{a(z)}{b(z)}$$

And a, b is holomorphic near z_0 and b has a simple zero at z_0.

$$b'(z_0) \neq 0$$

$$\text{Res}(f(z), z_0) = \lim_{z \to z_0} \frac{(z - z_0)a(z)}{b(z)} = \lim_{z \to z_0} \frac{a(z)}{\frac{b(z) - b(z_0)}{z - z_0}} = \frac{a(z_0)}{b'(z_0)}$$