

Computational Complex Analysis : : Class 05

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If the derivative exists for a point z_0 , then we have

$$\frac{\partial f}{\partial x} = \frac{1}{i} \frac{\partial f}{\partial y}$$

Suppose $f = f(z)$ has continuous partial derivatives with respect to x and y . Now we want to show f' exists. We know that f is differentiable in real variable sense.

$$f(z+h) = f(x+h_1, y+h_2) = f(x, y) + \frac{\partial f}{\partial x}(x, y)h_1 + \frac{\partial f}{\partial y}h_2 + \text{Smaller Terms}$$

Here $h = h_1 + ih_2$ Using this we can come to the proof,

$$\lim_{h \rightarrow 0} \frac{f(z+h) - f(z)}{h}$$

This thing exists. There is a proof for this. TODO.

Terminology

Terminology Frank Jones	Everyone except Frank Jones and his followers
Cauchy-Riemann Equation	Cauchy-Riemann Equations

Cauchy Riemann equation in polar form is

$$\frac{\partial f}{\partial r} = \frac{1}{ir} \frac{\partial f}{\partial \theta}$$

Problem 1. Show

$$\frac{d \log(z)}{dz} = \frac{1}{z}$$

$z \in \mathbb{Z}$