Computational Complex Analysis

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I could not be present in the class because there was time clash between the Weekly Jumah'r Namaz and 382, but here's a rough attempt to make this complete. This document will be updated with as many notes I can fetch from my friends and the office hours.

Problem 1.

$$\sinh(z+w)$$

Solution. What we will only use is $e^{z+w} = e^z e^w$. Using that you can show,

$$\sinh(z+w) = \cosh(z)\sinh(z) + \cosh(w)\sinh(z)$$

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□

What if we put weird $x + iy$ form inside of $\sin(x)$?

Problem 2. Explain $\sin(z)$.

Solution.

$$\begin{aligned}\sin z &= \sin(x+iy) \\ &= \sin x \cos(iy) + \cos(x)\sin(iy) \\ &= \sin(x)\cosh(y) + i\cos(x)\sinh(y).\end{aligned}$$

If we square this,

$$\begin{aligned}|\sin(z)|^2 &= \sin^2 z \cosh^2 y + \cos^2 x \sinh^2 y \\ &= \sin^2 z(\sinh^2 y + 1) + \sinh^2(1 - \sin^2 x) \\ &= \sin^2 x + \sinh^2 y.\end{aligned}$$

□

Now we will do some weird $\cos(\frac{2}{5})$ problem.

Problem 3. Solve $\cos 2\pi/5$.

Solution.

$$w = e^{2\pi i/5}$$

Hence, $w^5 = 1$. So,

$$\frac{w^5 - 1}{w - 1} = 0 = w^4 + w^3 + w + 1$$

Divide this by w^2,

$$0 = w^2 + w + 1 + \frac{1}{w} + \frac{1}{w^2} = \left(w + \frac{1}{w}\right)^2 + \left(w + \frac{1}{w}\right) - 1$$

From here,

$$w + \frac{1}{w} = e^{2\pi i/5} + e^{-2\pi i/5} = 2\cos\frac{2\pi}{5} = 2z$$

$$0 = 4z^2 + 2z - 1$$

Using the quadratic formula we get,

$$z = \frac{-1 \pm \sqrt{5}}{4}$$

□

Problem 4.

$$e^z = 1$$

Prove z is a integer multiple of 2.

Solution. I will have to do some reading on this. □