Honors Linear Algebra 354

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1 Office Hours

Tuesday, *12:30 - 2:30*. Wednesday *9:30 - 12:30* and *2:30 - 3:30*.

2 Vector Spaces

3 1.1 $\mathbb{R}^n$ and $\mathbb{C}^n$.

We can have a field $\mathbb{F}$ which can be either $\mathbb{R}$ or $\mathbb{C}$.
Let's talk a bout a List. A list can be,

$$(f_1, f_2, \ldots f_n)$$

These are ordered elements and the length of the list is n. But, it's no more a list if there are infinite elements.
$\mathbb{F}^n$ is the set of all lists of length n positive integer with elements of $\mathbb{F}$.

$$(x_1, x_2, x_3, \ldots x_n)$$

$\mathbb{R}^2$ can be drawn like in the cartesian plane, but it's difficult to draw $\mathbb{C}^2$. It is an ordered pair of complex numbers. These are vector spaces.

$$(x_1, \ldots x_n) + (y_1, \ldots y_n) = (x_1 + y_1, \ldots, x_n + y_n)$$

We can multiply scalar times vectors, note that a is any number. It can be real or complex.

$$a \in \mathbb{F}$$

For vectors v, u

$$a\,(v + u) = av + au$$

$$(a + b)\,v = av + bv$$

We define origin,

$$O = (0, 0, 0, 0, \ldots, 0)$$

We have a zero vector such that,

$$\vec{v} = \vec{v} + \vec{0}$$

4 Definition of Vector Space

Definition: There are objects called V vectors. There are scalars $\mathbb{F}$. Then we have operations which have the properties of either $\mathbb{R}^n$ or $\mathbb{C}^n$ made with no notation other than V.
We need addition, $\vec{v}+\vec{w}$. It's still vector. It's commutative $\vec{v}+\vec{w}=\vec{w}+\vec{v}$, its associative, $\vec{v}+(\vec{w}+\vec{z})=(\vec{v}+\vec{w})+\vec{z}$.
for all 0 we have $\vec{v}+0=v$. The scalar multiples, $a\in\mathbb{F}$ and $\vec{v}\in V$,

$$a\vec{v}\in V$$

We can have

$$0v=0$$

Proof:

$$(0+0)\,v=0v+0v=0v$$

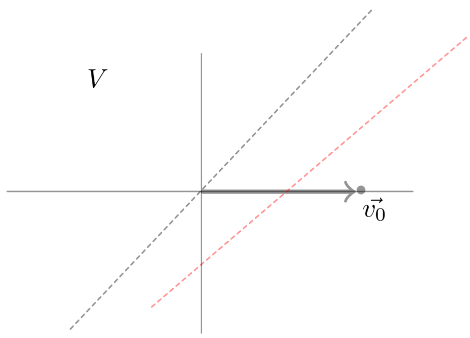
From here, subtracting one of the $0v$ from one side,

$$0=0v$$

A vector space with scalars $\mathbb{R}$ is called a real vector space. A vector space with scalars $\mathbb{C}$ is a complex vector space.
On P-16, in Sheldon Axler, V denotes (upper case V) always denotes a vector space over $\mathbb{F}$. And scalars can be complex or real, it's not always just simple $1,2,3,\ldots n$

5 Subspaces

Can a vector space be consist of just one element? Yes, element zero $\{0\}$ all by itself.
But can it be empty? Because it doesn't have the inverses. It just says it must have at least the origin. Definition: If V is a vector space. A subset U of V is said to be a subspace of V if it satisfies all the axioms of a vector space. Using structure of V itself.
$\vec{v_0}$ is a vector and the line that connects it can be a subspace. Any single line that goes through the origin in a $\mathbb{R}^2$ space is a subspace. Food for thought, can we prove or disapprove that for a space $\mathbb{R}^n$, $\mathbb{R}^{n-1}$ is a subspace.



Figuur 1: Vector space and subspace diagram, here Red is not a subspace because it doesn't have the origin.

6 Chapter 2: Finite Dimensional Vector Spaces

There is a name of this book. It was the textbook of the professor.
Let's review our standing assumptions,

- $\mathbb{F}$, V, here there is a field and V is a vector space over it.
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Learning objectives are,

- Span
- Linear Dependence
- Bases
- Dimension

2A span and linear independence.
Definition: A linear combination of a list $\vec{v_1}, \ldots, \vec{v_n}$ of vectors is any vector of the form,

$$a\vec{v_1} + \ldots + a_n\vec{v}_n$$

Here a_i is a scalar such that $a \in \mathbb{F}$. The set of all linear combination of the list $\vec{v}_1, \ldots, \vec{v}_n$ is a subspace of V.
This subset is a subspace of V, and called a span.
If span $\{\vec{v_1}, \ldots \vec{v_n}\} = V$, then we can say that $\vec{v}_1, \ldots \vec{v}_n$ span V.
For $\mathbb{R}^3$, span$\{(1,0,0),(0,1,0)\}$ is the $x - y$ plane. Until $z \neq 0$, we will have x, y plane to be the span for anything, because no matter how you make the linear combination, you cannot have any vector that points towards z to make a z span.
Definion: Linear Independence : Linear independent if $a_1\vec{v}_1 + a_2\vec{v}_2 + \ldots + a_n\vec{v}_n = 0$, can only and only be possible if you can use $a_1 = a_2 = a_3 = \ldots = a_n = 0$.

A list of one vector is linearly independent if and only if that vector is not 0. A list of two vectors is linearly independent if and only if neither are the scalar multiple of the other.

$$a_1\vec{v}_1 + a_2\vec{v}_2 = 0$$