Honors Multivariable Calculus : : Class 36

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Ahmed Saad Sabit, Rice University

Suppose $\vec{F}$ is defined on all $\mathbb{R}^2$ and curl $\vec{F}$ is 0 everywhere. Then

$$\oint_C \vec{F} \cdot \mathrm{d}\vec{s} = \int_R \mathrm{curl}\vec{F} = 0$$

We can conclude that $\oint$ is 0 then $\vec{F}$ is conservative.

Definition 1. A domain $D \subset \mathbb{R}^n$ is simply connected if it is path-connected and every loop in D can be "filled in" without leaving D.

Discussion on what is a simply connected loop is being done. It has to be some funny topological problem. Can we contract a loop into a single point without running into a domain issue? Well in $\mathbb{R}^3$ if we have a hole at $x, y, z = 0, 0, 0$ then a loop can be moved away from that point and contracted to 0.

Definition 2. A region $R \subset \mathbb{R}^2$ is a "simple region" (made up) if it can be described both as

$$R = \{(x, y) : a_1 \leq x \leq b_1, f_1(x) \leq y \leq f_2(x)\}$$

For some continuous functions f_1, f_2 satisfying $f_1(x) \leq f_2(x) \, \forall x \in [a_1, b_1]$. AND as

$$R = \{(x, y) : a_2 \leq y \leq b_2, g_1(y) \leq x \leq g_2(y)\}$$

For some continuous functions g_1, g_2 satisfying $g_1(y) \leq g_2(y) \, \forall y \in [a_2, b_2]$. Box, disk.

Vector Surface Integral

Let $\vec{F}$ in $\mathbb{R}^3$. The oriented surface be chosen S. Choose S to have one side and

$$\int_S \vec{F} \cdot \mathrm{d}\vec{S}$$

is integral of $\vec{F}$ over $\vec{S}$.

Simplest case is to consider a flat constant field. Take a plan and we have some surface S on that plane and it has a positive side.