

Honors Multivariable Calculus : : Class 08

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Speed is defined to be

$$s^2 = \vec{v} \cdot \vec{v}$$
$$\frac{d}{dt}s^2 = v \cdot v' + v' \cdot v = 2va$$

s is increasing if and only if $v \cdot a > 0$, s is decreasing if $v \cdot a < 0$. If it's constantly zero then we have no motion $s = \text{const.}$

Let's take pure circular motion,

$$p(t) = (\cos t, \sin t)$$

So we get

$$v(t) = (-\sin t, \cos t)$$
$$a(t) = (-\cos t, -\sin t)$$

Talking about $\vec{a} \cdot \vec{v}$ then

$$\vec{v} \cdot \vec{w} = |\vec{v}||\vec{w}| \cos \theta$$

Dot product being acute angle is the most olympiad shit out there haha. In general, one should try to something coordinate free as long as you can.

1 Thinking about Tangents

- If you keep zooming into a curve it becomes a line.
- Think about a water slide and you suddenly switch off gravity at some point.
- I am thinking $\lim_{\vec{x} \rightarrow \vec{a}}$ then tangent line is a line parallel to the vector $\vec{x} - \vec{a}$.